

Relativistic Deuteron Structure Function

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Abstract: We calculate the unpolarised deep inelastic structure function of a relativistic deuteron within a covariant framework. An exact treatment of nucleon off-shell effects is shown to give corrections to the widely-used convolution model, even in impulse approximation. Neglecting off-shell effects in the extraction of the neutron structure function from deuterium data introduces errors of order 1 – 2%.

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Because of its weak binding the deuteron is thought to be the ideal target from which to obtain information about the deep inelastic structure of the neutron. An accurate determination of neutron structure functions is vital for testing various parton model sum rules such as the Gottfried sum rule, which measures the flavour content of the proton sea, or the Bjorken sum rule in spin-dependent scattering. Recent precision experiments on unpolarised [1] and polarised [2] deuterium targets have enabled both of these to be tested more rigorously than ever before. However, the improved accuracy of the experiments has also meant that a definitive determination of these sum rules requires serious consideration of the nuclear corrections that arise when extracting neutron structure functions from the deuteron data. Considerable attention has already been focussed on nuclear shadowing [3, 4] and meson-exchange [3, 5] effects, which are typically a few percent below $x \sim 0.1$. On the other hand, the most obvious nuclear corrections are those arising from the fact that nucleons in the deuteron are bound, and thus off-mass-shell ($p^2 \neq M^2$). Despite this, a fully consistent treatment of off-shell effects in the deuteron is still outstanding.

The discussion of nucleon off-shell effects is usually based on some ansatz relating the deuteron structure function to a convolution involving the free nucleon structure function at a shifted value of x or Q^2 [6, 7, 8]. Instead, we present a systematic treatment of off-shell effects starting from the most general Dirac structure of the off-shell nucleon tensor and a covariant DNN vertex function. The general formalism was developed in Ref.[9]. Here we show how to extract the usual convolution from the full calculation — which is exact in impulse approximation (c.f. Ref.[10]). The practical importance of the approximations needed to arrive at the convolution formula is estimated within a specific model for the structure of the nucleon and the DNN vertex.

As shown by Melnitchouk et al. [9], in the Bjorken limit the most general form of the truncated (off-shell) nucleon structure function is $\chi(p, q) \equiv I \chi_0 + \not{p} \chi_1 + \not{q} \chi_2$, where χ_i are real functions of p and q . To calculate the complete deuteron structure function one must also describe the off-shell nucleon—deuteron interaction, which we write in the form: $\mathcal{A}(P, p) \equiv I \mathcal{A}_0(P, p) + \gamma^\alpha \mathcal{A}_{1\alpha}(P, p)$. Taking the trace of $\mathcal{A}(P, p) \otimes \chi(p, q)$ and integrating over p gives the spin-averaged quark distribution per nucleon in the deuteron:

$$q^D(x) = \frac{1}{2\pi^2} \int dy dp^2 (\mathcal{A}_0 \chi_0 + \mathcal{A}_1 \cdot p \chi_1 + \mathcal{A}_1 \cdot q \chi_2) \quad (1)$$

where $x = -q^2/2P \cdot q$ is the Bjorken scaling variable, and $y = p \cdot q/P \cdot q$ is the fraction of the light-cone momentum of the deuteron carried by the nucleon. In impulse approximation the deuteron structure function involves an on-shell spectator while the off-shell nucleon is

struck. The DNN vertex of Buck and Gross [11] is therefore ideal for this purpose. For a deuteron at rest, it relates \mathcal{A}_0 and $\mathcal{A}_{1\alpha}$ (which include the off-shell nucleon propagators) to the relativistic deuteron wavefunctions:

$$\begin{aligned}\mathcal{A}_0 &= 2M_D\pi^2 M \{ \mathcal{C} - 2\mathcal{P} + \mathcal{D} \} \\ \mathcal{A}_{1\alpha} &= 2M_D\pi^2 \left\{ \left(p_\alpha - \frac{p^2 - M^2}{M_D^2} P_\alpha \right) \mathcal{C} \right. \\ &\quad \left. + \left(-p_\alpha + \frac{M_D^2 + p^2 - M^2}{2M_D^2} P_\alpha \right) \left(2\mathcal{P} + \frac{M^2}{\mathbf{p}^2} \mathcal{D} \right) \right\}\end{aligned}\quad (2)$$

where

$$\begin{aligned}\mathcal{C} &= u^2(|\mathbf{p}|) + w^2(|\mathbf{p}|) + v_t^2(|\mathbf{p}|) + v_s^2(|\mathbf{p}|) \\ \mathcal{P} &= v_t^2(|\mathbf{p}|) + v_s^2(|\mathbf{p}|) \\ \mathcal{D} &= \frac{2|\mathbf{p}|}{\sqrt{3}M} \left[u(|\mathbf{p}|) \left(v_s(|\mathbf{p}|) - \sqrt{2}v_t(|\mathbf{p}|) \right) + w(|\mathbf{p}|) \left(v_t(|\mathbf{p}|) + \sqrt{2}v_s(|\mathbf{p}|) \right) \right].\end{aligned}\quad (3)$$

The functions u and w correspond to the usual non-relativistic S - and D -state deuteron wavefunctions, while v_s and v_t represent the small singlet and triplet P -state wavefunctions which are purely relativistic in origin. In our calculation we use the wavefunctions from Ref.[11] with pseudoscalar π exchange, although similar results are obtained with wavefunctions derived from a model with pseudovector πNN couplings. (The momentum \mathbf{p} is calculated as in Eq.(7) below.)

Of particular interest is the connection between Eq.(1) and the usual convolution formula [6, 7, 8], in which the nuclear structure function is a one-dimensional convolution of the momentum distribution of a bound nucleon in the deuteron ($\varphi(y)$ in Eq.(5) below) with the on-shell nucleon structure function ($q^N(x/y)$). In terms of the truncated nucleon functions the latter is defined as:

$$q^N(x/y) = 4M \chi_0^{on} + 4M^2 \chi_1^{on} + 4p \cdot q \chi_2^{on} \quad (4)$$

where χ_i^{on} are obtained by setting $p^2 = M^2$ and $p_T = 0$ in the fully off-shell χ_i . In order to derive such a convolution formula one needs to factorise $q^N(x/y)$ from the integrand in Eq.(1). This in turn requires that either the functions χ_i are proportional to each other, or two of them equal to zero. Alternatively, the χ and \mathcal{A} parts of Eq.(1) could be factored if the functions \mathcal{A}_i satisfied $\mathcal{A}_0/M = \mathcal{A}_1 \cdot p/M^2 = \mathcal{A}_1 \cdot q/p \cdot q$. From Eqs.(2) and (3) it is evident that this condition is fulfilled in the $p^2 = M^2$ limit by the term proportional to \mathcal{C} . By further separating χ into on- and off-shell parts, Eq.(1) can be recast in the form:

$$q^D(x) = \int_x^1 \frac{dy}{y} \varphi(y) q^N(x/y) + \delta^{(A)} q^D(x) + \delta^{(x)} q^D(x) \quad (5)$$

where the first term is the usual convolution result, in which q^N is given by Eq.(4), and

$$\varphi(y) = \frac{M_D}{4} y \int_{-\infty}^{p_{max}^2} dp^2 \frac{E_p}{p_0} \mathcal{C}(p). \quad (6)$$

Here the energy E_p is

$$E_p = \sqrt{M^2 + \mathbf{p}^2} = \frac{M^2 - p^2 + M_D^2}{2M_D} \quad (7)$$

in the deuteron rest frame, and $p_{max}^2 = yM_D^2 - yM^2/(1-y)$ is the maximum kinematic value of p^2 .

The two correction terms in Eq.(5) can be identified with the off-shell and P -state components of the relativistic DNN vertex,

$$\begin{aligned} \delta^{(A)} q^D(x) &= \frac{M_D}{2} \int_x^1 dy \int_{-\infty}^{p_{max}^2} dp^2 \left\{ \left[\frac{1}{2} \left(1 - \frac{E_p}{p_0} \right) q^N(x/y) \right. \right. \\ &\quad \left. \left. + \left(\frac{E_p}{M_D} \chi_1^{on} - \frac{P \cdot q}{M_D^2} \chi_2^{on} \right) (p^2 - M^2) \right] \mathcal{C} \right. \\ &\quad \left. + \left[-2M \chi_0^{on} + 2\mathbf{p}^2 \chi_1^{on} + \left(1 - y - \frac{E_p}{M_D} \right) P \cdot q \chi_2^{on} \right] \mathcal{P} \right. \\ &\quad \left. + \left[M \chi_0^{on} + M^2 \chi_1^{on} + \frac{M^2}{\mathbf{p}^2} \left(1 - y - \frac{E_p}{M_D} \right) P \cdot q \chi_2^{on} \right] \mathcal{D} \right\} \end{aligned} \quad (8)$$

and with the off-shell part of the truncated nucleon structure function,

$$\delta^{(\chi)} q^D(x) = \frac{1}{2\pi^2} \int dy dp^2 \left(\mathcal{A}_0 \chi_0^{off} + \mathcal{A}_1 \cdot p \chi_1^{off} + \mathcal{A}_1 \cdot q \chi_2^{off} \right) \quad (9)$$

where $\chi_i^{off} \equiv \chi_i - \chi_i^{on}$.

We should note that the convolution term which we have identified is not unique. An alternative definition of $\varphi(y)$ in terms of $\mathcal{C} - \mathcal{P}$, with an analogous redefinition of the $\delta^{(A)} q^D$ term, can also produce the required proportionality of \mathcal{A}_0 and $\mathcal{A}_{1\alpha}$ needed to satisfy the convolution criterion. Nevertheless Eq.(6) is the most natural choice and one that has been followed in many phenomenological treatments. In particular, $\varphi(y)$ includes the infamous relativistic flux factor, $(E_p/p_0) y$ [12], and therefore satisfies the normalisation condition

$$\int_0^1 dy \varphi(y) = 1. \quad (10)$$

This condition is necessary in order that (for the valence component) the convolution term alone preserves the baryon number of the deuteron. Since both of the correction terms in Eq.(5) are proportional to either $(p^2 - M^2)$ or the P -state wavefunctions, we expect the $\delta^{(A)} q^D$ and $\delta^{(\chi)} q^D$ terms to be small. However, to obtain a quantitative estimate of these corrections requires a model for the off-shell nucleon functions χ_i .

Our model for the nucleon structure function is motivated by the work of Refs.[13, 14, 15]. In particular, we suppose that at some relatively low scale ($Q_0 \sim 400$ MeV) the nucleon is well described in terms of its valence quarks. Then, as explained in Ref.[9], we can calculate the χ_i in terms of a set of phenomenological vertex functions, $\Phi^{(S)}(p, k)$, describing the process $N \rightarrow q(qq)$, where the (qq) -pair has spin $S = 0$ or 1 . On the basis of bag or constituent quark models we expect these diquark states to have masses $m_S \simeq 700$ (900) MeV for $S = 0$ (1) [14]. For simplicity we choose a single function for each type of vertex, say $I\Phi^{(0)}$ and $\gamma_5\gamma_\alpha\Phi^{(1)}$, and parameterise their momentum dependence in the form

$$\Phi^{(S)}(p, k) = N(p^2) \frac{k^2}{(k^2 - \Lambda_S^2)^{n_S}}. \quad (11)$$

With the above values for m_S (note that as the spectator is on-mass-shell, $m_S^2 = (p - k)^2$) the cut-offs, Λ_S , and exponents, n_S , are chosen to fit the on-shell data. The function $N(p^2)$ parameterises the nucleon off-shell dependence of the vertex functions. For the deuteron, because of the strong peak in the p_T distribution at small transverse momentum ($p_T \sim 25$ MeV), $N(p^2)$ is well approximated by a constant.

We could equally well choose to parameterise the distributions at larger Q_0^2 , however the identification of m_S with diquark masses would be lost, as would the use of approximate SU(4) symmetry to relate the spin-flavour distributions to the valence quark distributions (namely $d_{val} = q_{val}^{(1)}$ and $u_{val} = (q_{val}^{(1)} + 3q_{val}^{(0)})/2$, where $q_{val}^{(0)}$ and $q_{val}^{(1)}$ are the distributions with $S=0$ and $S=1$ diquark spectators, each with its first moment normalised to unity). Convergence of the k^2 integrations in the functions χ_i imposes constraints on the exponents n_S : $n_0 > 1$, $n_1 > 1.5$. The correct valence d/u ratio at large x also requires that $n_1 \approx n_0 + 1$. The values $n_{0(1)} = 1.3$ (2.4) are found to reproduce the large- x data well when evolved to $Q^2 \sim 5$ GeV². The other two parameters, the cut-offs $\Lambda_{0(1)}$, are 750 (450) MeV. The fact that $\Lambda_0 < \Lambda_1$ can be understood in a constituent quark picture from the smaller radius of the scalar two-quark system.

For the sea component of the nucleon we take a similar form for the vertex functions, but assume that sea quark distributions are associated with intermediate spectator states having somewhat larger masses, $\sim 2m_{0,1}^{val}$. (In bag model calculations [13] the sea corresponds roughly to four partons in the intermediate state, compared with two for the valence quarks.) To suppress the large- k_T components of the vertex functions for the sea we choose $n_{0(1)} = 4$ (5), which, with cut-offs $\Lambda_{0(1)} = 0.7$ (0.7) GeV, give the observed rapid fall-off at large x . In addition, we parameterise the observed flavour asymmetry in the proton sea by taking $d_{sea} - u_{sea} = 2(1 - r)q_{sea}$ ($0 < r < 1$), subject to the constraint $\int_0^1 dx (d_{sea} - u_{sea}) \approx 0.11$

[1], where $q_{sea} \equiv (u_{sea} + d_{sea})/2$.

Although at the quark model scale, Q_0 , the low resolution means that valence quarks dominate, the work of Glück et al. [15] suggests the phenomenological need of a small amount of glue. For the shape of the input gluon distribution we use their valence-like parameterisation: $xg(x) \sim x^2(1-x)^4$. With the above parameters the second moments of $(u+d)_{val}$ and q_{sea} are $\approx 83\%$ and 3% , respectively, which is sufficient to saturate the momentum sum rule. The smaller fraction of momentum carried by gluons here compared with the analysis of Ref.[15] reflects the slightly smaller value of $Q_0^2 (= (0.39 \text{ GeV})^2)$ in our fit, and the rapid rise of $\langle x \rangle_g$ with Q^2 . In Fig.2 we show the resulting proton structure function, $F_{2p} = x(4u_{val} + d_{val})/9 + x(6r + 4)q_{sea}/9$ (with $r \approx 0.91$), evolved to $Q^2 = 5 \text{ GeV}^2$. Clearly the data is very well described over the entire range of x .

Without introducing any other parameters we can now calculate the deuteron structure function, $F_{2D} = 5x(u_{val}^D + d_{val}^D)/9 + 20xq_{sea}^D/9$. The result, evolved to $Q^2 = 5 \text{ GeV}^2$, is also shown in Fig.2. Here $q_{sea}^D \equiv (u_{sea}^D + d_{sea}^D)/2$, and the flavour distributions q^D are defined to be those for a bound proton in the deuteron, with charge symmetry assumed for the neutron distributions (c.f. Ref.[16]). The agreement with recent data from SLAC, (reanalysed) EMC-NA2 and NMC F_{2D} data [17] is clearly excellent.

In order to exhibit the effect of binding and Fermi motion, in Fig.3 we show the ratio of F_{2D} to $F_{2N} (= F_{2p} + F_{2n})$ as a function of the variable $x_N \equiv 2x$. It shows the same characteristic features observed in the nuclear EMC effect for heavy nuclei [18]. The combined nuclear effect in deuterium, due to binding, Fermi motion and nucleon off-shellness, is predicted to be about 5% at $x_N \simeq 0.6-0.7$, which happens to be similar to that found in Ref.[19]. (Reliable predictions for the ratio below $x_N \sim 0.2$ would require inclusion of additional mechanisms beyond the impulse approximation considered here.)

Having obtained a good fit for the proton and deuteron structure functions we are now in a position to test the numerical importance of the off-shell corrections in Eq.(5). In Fig.4 we plot the ratios of (the valence components of) the individual terms to the total F_{2D} . Both $q^D(x)$ and the convolution component in Eq.(5) are normalised to 1 by choosing $N(p^2)$ in Eq.(11) to be a constant, about 0.7% (2.6%) larger for the $S = 0$ (1) vertex than the corresponding normalisation constant for an on-shell nucleon. As a result the first moments of $\delta^{(A)}q^D$ (negative) and $\delta^{(X)}q^D$ (positive) cancel. The ratio of the convolution component to the total deviates from unity by about 1-2% for $x_N < 0.9$. (In fact, for $x_N > 1$ the off-shell

corrections start to dominate.) Although the off-shell effects are numerically small, in any precision analysis of the neutron structure function all nuclear effects should be included.

In summary, we have calculated the complete relativistic deuteron structure function within a covariant formalism, including the effects due to binding, Fermi motion and nucleon virtuality. The off-shell effects associated with the nucleon structure function as well as with the DNN vertex function are found to give corrections to the usual convolution formula. Consequently the common deconvolution procedure [7, 20] of extracting the neutron structure function from deuterium data will introduce errors in F_{2n} of $\approx 1 - 2\%$ for $x_N < 0.9$. Off-shell effects also give rise to corrections to the convolution formula for the polarised deuteron structure function g_{1D} [2], and the consequences of these for g_{1n} are currently under investigation [21].

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Figure Captions.

1. Deep inelastic scattering from a deuteron in the impulse approximation. The respective photon (q), quark (k), nucleon (p) and deuteron (P) momenta are indicated.
2. Total proton and deuteron F_2 structure functions, calculated and evolved to $Q^2 = 5$ GeV^2 . The data are a compilation of the SLAC, re-analysed EMC-NA2 and NMC data [17] for $4 < Q^2 < 6$ GeV^2 .
3. Ratio of the deuteron and isoscalar nucleon structure functions in the valence dominated region at $Q^2 = 5$ GeV^2 .
4. Off-shell nucleon and DNN vertex corrections, $\delta^{(x)}q^D$ (dotted) and $\delta^{(\mathcal{A})}q^D$ (dashed), respectively, as a ratio to the total q^D (see Eq.(5)). The solid curve is (convolution/total) -1 . Note that even though the first moments of $\delta^{(x)}q^D$ and $\delta^{(\mathcal{A})}q^D$ are equal in magnitude, because these are divided by the x -dependent q^D , the areas under the dashed and dotted curves are not equal.







